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The Influence of Positivism on the Teaching

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This article describes the strong influence of positivism on the teaching of mathematics in Brazil. The dissemination of positivism occurred in a very intensive way from 1870 to 1930, due mainly to the strong leadership of teachers at the military and engineering academies. From its firmly entrenched position in these institutions, the positivistic ideology affected the social, political, pedagogical, and ideological life in Brazil. Here, I identify the main representatives of positivism, who focused their research on Auguste Comte's concept of mathematics. They oriented curricula and programs according to Comte's principles as well as produced mathematics with a distinct positivist bent. Although a marked decline occurred after 1930, the positivistic phenomenon was not exhausted as a research topic, and, indeed, it still has not been entirely extinguished in Brazilian life. © 1999 Academic Press

Este trabalho descreve a forte influência do positivismo no ensino da Matemática no Brasil. A difusão do positivismo aconteceu de forma muita intensa entre 1870 e 1930, devido principalmente a atuação dos docentes-militares, que mantinham uma liderança forte nas academias militares e de engenharia. Nestas instituições a ideologia positivista encontrou uma forte sustentação e pode, então, ter efeitos na vida social, política, pedagógica e ideológica brasileira. Identificamos os principais representantes do positivismo no círculo acadêmico. Detectamos as primeiras manifestações da concepção de Matemática de Auguste Comte em livros-texto. Identificamos a orientação de currículos e programas segundo os preceitos de Comte e analisamos principalmente as obras de Matemática de autores positivistas. O declínio do positivismos depois de 1930 também é registrado. O fenômeno positivismo não foi esgotado como tema de pesquisa e tudo indica que ainda não se extinguiu completamente da vida brasileira. © 1999 Academic Press

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SCIENTISM AND THE ARRIVAL OF POSITIVISM IN BRAZIL

Positivism was an ideology adopted in the 19th century by the South American countries in an effort to facilitate their entry into the modern world and was especially strong in Brazil. During the colonial period (1500–1800), Brazilian society consisted of two groups: owners of large tracts of land and slaves. In the second half of the 18th century, a middle class composed of traders and mine owners began to emerge [28]. Until 1808, due to the absence of institutions of higher education in Brazil, the ruling upper class and the emergent middle class tended to send their children to study in Europe, mainly to the University of Coimbra in Portugal [72]. After a reform in 1772, the University of Coimbra revised its courses, especially in science, and created two new faculties: mathematics and philosophy (or "natural sciences" in moderns terms) [19; 35; 71]. Sebastião José de Carvalho e Mello (1699–1782), the "Marquis of Pombal," played a fundamental role in this reform. A graduate of reformed

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Coimbra, the Bishop Azeredo Coutinho opened the Seminary of Olinda in Pernambuco during the Brazilian colonial period. It became Brazil's first institution of modern teaching, emphasizing mathematics as well as the physical and natural sciences [55; 68].

In Portugal, prepositivistic ideas were spread mainly by the Marquis of Pombal [69] and by Luis Antonio Verney (1713–1792). Their views inspired the reform of the University of Coimbra [12; 13] and can be characterized both as a break from the ancient system and as a renewal of studies. The Marquis of Pombal remains an enigma to historians, although they agree that he adopted an anticlerical stance and was strongly influenced by illuminism as well as by English empiricism. He formulated a concept of teaching that can be characterized as part of 18th-century prepositivism. Similarly, in his book, *The True Study of Method*, Verney embraced the conception of reform [68, 54].

Mathematics teaching in 19th-century Brazil reflected two principal influences. During the colonial period and at the beginning of the monarchy (early 19th century), Portuguese thought dominated, but later, French ideas came to the fore. Both had strong links with the Enlightenment and with the prepositivism of the 18th century [26]. The principal representatives of prepositivism fall roughly into two generations: Étienne Condillac (1715–1780), Jean D'Alembert (1717–1783), and Anne-Robert Turgot (1721–1788) in the first, and Joseph Louis Lagrange (1736–1813) and Antoine Lavoisier (1743–1794) in the second. One root of this philosophy is to be found in the thought of John Locke (1632–1704). Locke, the real founder of English empiricism, held that all ideas come from experiences, and these come in two forms, external sensations and reflections [41]. Locke's ideas strongly influenced such Enlightenment thinkers as Condillac, while the Cartesian tradition of French algebraic analysis (represented by D'Alembert and Lagrange) shaped the 18th-century French roots of positivism [68]. The philosophical conceptions of Turgot and D'Alembert relative to mathematics and the natural sciences may be seen as precursors of Comte's positivism [50, 11].

In Portugal, the Marquis of Pombal—influenced by these ideas—aimed to transform a country, essentially made up of an aristocracy and a class of agriculture workers, into a strong commercial state run by a new class of well-educated public officials [68]. During Pombal's government, conflict between France and Spain constituted a menace to Portugal's security. Pombal needed England's help to protect Portugal militarily, but in accepting this help, he did not want to fall prey to English dominance. By pursuing internal economic and social reforms, Pombal sought to strengthen his country from within and, in so doing, make it less vulnerable to outside economic and cultural forces. Education played a key role in his plan, and mathematics played a key role in his conception of education.

Both the Marquis of Pombal and Verney viewed mathematics as essential to all disciplines, including law and theology [19]. They shared this conception of the centrality of mathematics with the French prepositivists, especially D'Alembert and Lagrange, as well as with Comte himself [30; 31; 32; 68; 83]. Pombal's reform played an important role in Brazil, for it guided the establishment of institutional bases that allowed Comte's positivism to develop and spread [68; 72]. In particular, the Marquis's reform echoed in Brazil with the foundation in 1810 of the Military Academy of Rio de Janeiro and its teaching devoted to the relations between theory and practice [69]. This institution became a source for disseminating the new way of thinking and laid the groundwork for Comte's new utilitarian and positivist thought.

Education in Brazil, as pointed out by historian Primitivo Moacyr, was the way to good and not to general culture [51, 5]. During João VI's government (1808–1820), for example, several venues for higher learning were created, such as courses in surgery, military academies, a standing army, public libraries, and the botanical garden. Nevertheless, none of these was developed as part of a global plan for teaching in the country, and the regency period (1831–1840) did not witness such a plan [68; 72]. Institutions of higher education were created, but secondary schools did not exist. In 1837, for instance, the *Colégio Pedro II* was founded; after 6 years of study there, students earned the degree of bachelor of arts [72].[1] Its curriculum revealed a tendency to universalistic and encyclopedic teaching. Mathematics was initially taught during the 6-year course of study and comprised arithmetic, algebra, and geometry. Although the amount of mathematics taught gradually decreased, the curriculum of the *Colégio Pedro II* served as a model for schools created in other provinces. The foundation of the first *Escola normal* in Rio de Janeiro in 1835, however, actually predated that of the *Colégio Pedro II*.

The lengthy reign of Pedro II (1840–1889), the so-called empire period, witnessed few cultural changes in Brazil. Literature and literary discussion may be said to have somewhat predominated over scientific activities [82].

THE FIRST MANIFESTATIONS OF POSITIVISM IN BRAZIL

Auguste Comte's positive philosophy, which I consider 19th-century positivism, emerged in Paris in 1830 [27]. Comte's positivism had a strong influence in several areas of Brazilian society. With the beginning of doctoral studies in mathematics at the military academy, the *Escola Central*, in 1849, the first references to Comte were made in theses that tackled themes related to applied mathematics, a subject of interest to engineers.[2] These first manifestations of positivism, however, are limited to mentioning Comte's name or his work [74]; they do not yet represent a wide propagation of positivistic thought. From 1851 to 1887, 24 theses were produced, but only 6 of them contained some sort of reference to Comte. The most important period for positivism in Brazil started in the 1870s and extended into the 1910s [68].

From 1832 to 1842, some Brazilians—like José de Almeida, Patricio d'Almeida e Silva, Agostinho Roiz Cunha, Antonio Campos Belos, Felipe de Araújo Pinho, and Antonio Machado Dias—attended free, private classes taught by Comte in Paris. Felipe de Araújo Pinho, in fact, was awarded a bachelor's degree in mathematics by the Faculty of Sciences of the University of Paris in 1842 [40].

In 1858, a book, entitled *Elements of Mathematics*, by Antonio Araújo Ferrão Muniz de Aragão (1813–1887) appeared in the Brazilian province of Bahia. It was the first book published in Brazil that dealt with positivistic mathematics, and it provided an extensive discussion of Comte's philosophy in its 40-page introduction [8]. Aragão had attended courses in London and Paris and, beginning in 1836, had propagated positivistic ideas in Bahia where he taught mathematics. Aragão's objective, declared in his book's introduction,

[1] Only the military academy awarded bachelor's degrees in the physical sciences and mathematics.

[2] See, for example, the 1851 "Dissertation on the Fundamental Principles of Floating Bodies Dipped in Two Resistant Fluids and about Stability in Naval Construction" and the thesis in 1855 on "General Equations of Heat Diffusion in Solid Bodies, Supposing Conductivity Variable with Direction and Position."

was "to present mathematics from a philosophical point of view. Method will be taken into especial account, but the important principles will also be taken into consideration" [8, xiv].

Aragão considered mathematics a useful science because: (1) its object is simple, (2) its principles are clear and evident, (3) it is fundamentally defined, (4) its demonstrations are rigorous and logical, and (5) it is a verifiable science. As did Comte, Aragão viewed mathematical study as exemplary and instrumental [8; 26; 27]. Mathematics was the basis for all other kinds of studies. In his book, Aragão tried to present arithmetic from a rigorous point of view, stating axioms, definitions, and theorems. His objective was to demonstrate all possible combinations using the notions of quantity and number. He stated the fundamental principles, characterizing arithmetic as a deductive science.

According to Aragão, the sciences are nothing more than collections of facts. The definitions of numbers are, therefore, arithmetic facts obtained through observation, while axioms are fundamental truths, obtained by induction, that serve to justify theorems. After presenting general considerations about nomenclature, he enunciated four arithmetic axioms (that can also be found in Euclid's *Elements*):

> 1st axiom: The whole is larger than each of the parts.
>
> 2nd axiom: The whole equals all its parts taken together.
>
> 3rd axiom: Quantities that are equal to the same quantity are equal to each other, or, in more general terms, things equal to the same thing are equal one another. If a quantity A and B are, each one equal to the quantity Z, A is equal to B, or if $A = Z$ and $B = Z$, then $A = B$.
>
> 4th axiom: If to equal quantities we add equal quantities, or, in general terms, equals added to equals form equals. If A is equal to B, and $a = b$, the sum $a + A = b + B$. [8, 23][3]

Aragão stated no other arithmetic axioms. In his view, many simple truths that are normally considered axioms can be deduced from these four axioms such as if out of equals we take equals, equals remain. This type of discussion usually did not appear in contemporary arithmetic textbooks. It is thus interesting to notice Aragão's concern for a formal treatment of arithmetic [8, 44].

THE DIFFUSION OF POSITIVISM IN MILITARY AND ENGINEERING SCHOOLS

Many teachers at the *Escola central*, and later at the *Escola politécnica*, publicly declared themselves to be positivists [56]. The influence of their ideas on their students was notable, as shown in the work of historian Ivan Lins [40]. Moreover, these positivists had a marked effect on the society of that time. By way of illustration, consider some of the positivists who played important roles in public life as engineers and politicians.

Americo Viveiros, Saturnino de Brito, and João Batista all espoused positivist ideals as a result of attending the *Escola politécnica* in Rio de Janeiro. Viveiros, born in the state of Maranhão, graduated from the school in 1886 and went on to pioneer industrial chemistry in Brazil. Brito, born in Rio Grande do Sul, graduated a year later and was considered the most outstanding sanitary engineer in the country [40; 53; 65]. Together with Aarão Reis,

[3] 1° axioma: O todo é maior que cada uma de suas partes. 2° axioma: O todo é igual a todas as suas partes tomadas juntas. 3° axioma: As quantidades iguais a mesma quantidade são iguais às outras, ou em termos mais gerais, coisas iguais à mesma coisa são iguais umas às outras. Se a quantidade A e B são cada uma igual a quantidade Z, A é igual à B , ou se $A = Z$ e $B = Z$ então $A = B$. 4° axioma: Se à quantidades iguais ajuntamos quantidades iguais, ou em termos mais gerais, iguais ajuntados à iguais formam somas iguais. Se A é igual à B, e $a = b$, as somas $a + A = b + B$.

he participated in the construction of the city of Belo Horizonte, the capital of the state of Minas Gerais [42]; in São Paulo and Recife, Brito worked to construct drains and a sanitary system. Finally, Batista, born in Piaui, went on to build railways after his graduation in addition to work as an engineer in Rio Grande do Norte, where he sought to solve the state's chronic drought problem [40].

On the professorial side, Carlos Sampaio, Alvaro de Oliveira, Aarão Reis, Francisco Braga, João Felipe Pereira, and Licinio Athanasio Cardoso all taught within the positivist framework of the *Escola Politécnica*. Sampaio, who also lectured at the *Escola naval* in Rio de Janeiro, was mayor of the city during the government (1919–1922) of Epitacio Pessoa. Alvaro de Oliveira, one of the founders in 1876 of the Positivistic Association (*Apostolado positivista*) of Rio de Janeiro, taught chemistry at the *Escola*, while Aarão Reis, an outstanding civil engineer and one of the best Brazilian technicians in railroad construction, taught economic policy there. The latter authored several books, among them, *Elementary Course of Mathematics* [62; 63; 64]. Braga, on the other hand, taught the preparatory course at the *Escola*, beginning his lectures by stating the "laws of positivist philosophy." In 1901, he became a teacher of differential and integral calculus [40]. Finally, Pereira lectured on hydraulics and served as a minister in the government (1891–1894) of Floriano Peixoto, while Cardoso taught calculus and played an important role in the professionalization of mathematics in Brazil (see below).

Internationally, Ernesto Otero, born in the state of Rio Grande do Sul, attended the *Technische Hochschule* in Karlsruhe, Germany, where he earned his doctoral degree. In Brazil, he dedicated himself to the construction of railways, mainly in Rio Grande do Sul [40]. He associated with the positivist politician Julio de Castilhos and with Demetrio Ribeiro, and gave financial support for positivistic mathematical publications such as the book, *Arithmetic* (1906), by Manuel Almeida Cavalcanti.

In the state of Rio Grande do Sul, where the army had always had one of its larger military bases, the *Escola militar* of Porto Alegre, founded in 1851, was another focal point for the spread of positivist ideas [40; 51]. This military preparatory school for students going on to artillery school in Rio de Janeiro offered basic courses in mathematics. In 1877, the one-year preparatory course included algebra and differential and integral calculus. Joaquim Salles Homem, one of the most active people in the province, was one of the directors of the *Escola militar*, and also wrote a textbook, entitled *Handbook of School Philosophy* (1889), that widely incorporated Comte's ideas [45]. All of these men fully embraced positivism as it was practiced and preached at their respective institutions, and especially at the *Escola politécnica* in Rio de Janeiro, at the close of the 19th century.

BENJAMIN CONSTANT MAGALHÃES: EXPOUNDER OF POSITIVISM

In Brazil, the positivists fell into two distinct groups, the orthodox and the heterodox. The orthodox group accepted Comte's so-called religion of humanity and included Teixeira Mendes, Miguel Lemos, and Licinio Cardoso, all proponents of mathematics based on Comte's book, *Subjective Synthesis* (1856) [68; 70]. The heterodox group included Benjamin Constant Magalhães and Roberto Trompowsky Almeida, both of whom followed mathematics according to Comte's *Course of Positive Philosophy* (1830–1840) and *Analytic Geometry* (1843).

FIG. 1. Chapelle de l'Humanité, 5 Rue Payenne, Paris.

Comte's ideas continued to spread among Brazilian intellectuals, mainly at military, polytechnic, and naval schools. In 1876, a group of teachers, engineers, and military men founded the *Apostolado positivista do Brasil* [40]. In addition to Magalhães, Miguel Lemos and Teixeira Mendes also joined this group. Later, the association, which changed its name

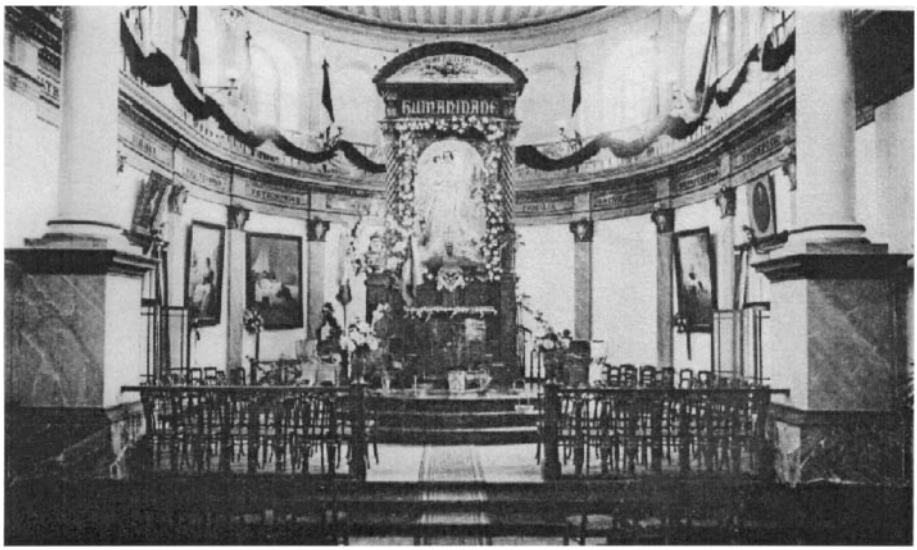

FIG. 2. Igreja da Humanidade no Rio de Janeiro.

to the Positivistical Apostolate of Brazil, adopted a religious outlook, and Magalhães, due to disagreements with Lemos, left it. In truth, Magalhães never agreed with Comte's religious ideal, but he always propagated the positivist philosophy [46].

Benjamin Constant Botelho Magalhães, the principal expounder of positivism in Brazil, was born at Niterói in 1837. His education began under his father, who was a school teacher. Without financial support to pay for his education and in order to provide monetary support for his family, Magalhães entered into a military career in 1852. He had his first contact with Comte's ideas in 1857, when he read the first volume of Comte's *Course of Positive Philosophy*. Two years later, he became the mathematics examiner of candidates for higher courses in the empire. He remained in this post until 1876 [11; 46; 70].

In 1860, Magalhães earned a bachelor's degree in the physical and mathematics sciences. Although he obtained the highest mark of all those competing on the professional entrance examination for a teaching position at the *Colégio Pedro II*, the second-ranked candidate was appointed to the position, clear evidence of the corruption of the system. In 1863, Magalhães was named mathematics teacher at the *Instituto dos Cegos*, and 6 years later he became the director of that institute [46; 70].

In 1868, after spending one year fighting in the war between Brazil and Paraguay (1864–1870), Magalhães wrote a textbook, entitled *Theory of Negative Quantities* [43; 70]. By 1873, he had taken the examination to enable him to become a lecturer in mathematics at the *Escola militar* and had taken first place. From this position he affirmed his adherence to positivism and made it known that he would use it as a guide to teaching. Because of Magalhães's influence, Comte's philosophy became well known to the former's students during the years 1881–1889 when preparations were being laid for the proclamation of the Brazilian republic in 1889 [68]. Magalhães was very popular among his students as contemporary statements reveal: "What a bright mathematics staff our Escola Militar had! Among the teachers, Benjamin Constant stood out. It seemed that the subject had no secrets from him; he dominated it with a rare mastery. Instead of a monotonous sequence of axioms, theorems, and corollaries, we listened with true delight to a synthetic exposition of the subject, over which he flew, transporting us with the magic of his convincing thoughts and his refined speech. His lectures were further brightened by a sweet physiognomy, which captivated and truly fascinated us" [40, 316].

Little has been said thus far about the remarkable connection between the rise of the middle class and the army [52]. From the time of the war between Brazil and Paraguay, the army gained both internal cohesion and stability in society, choosing its officials from the developing middle class. It is, therefore, natural that the army served as counsel, in some sense, to the republican movement. The monarchy, on the other hand, represented the interests of large landowners and of exporters and importers [38; 56].

Due to the development of coffee agriculture, manual workers began to appear, and São Paulo became a dynamic region in the Brazilian economy [38]. For this group to become stronger politically, it needed the support of other nonruling classes, among them, the militants of "republican propaganda,"[4] the civil and military positivistic circles, middle class public officials, and liberal professionals. The military and the middle class, which took a radical position relative to the decline of the monarchy, worked for greater political participation [28].

[4] This group was formed to promote the establishment of the republic of Brazil.

In 1872, the Republican party was founded in São Paulo. Its platform included religious freedom and the separation of church and state. Freemasonry had also grown significantly, with the most influential intellectuals including both masons and Catholics [55]. This republican alliance strengthened its relationship with the military as well. In 1871, the *Clube militar* was founded to promote the interests of the military; 6 years later, it had become a very strong political force [40].

Magalhães played an active role in the proclamation of the republic in November 1889, as the leader of young officials. After the proclamation, he was named Minister of War, but Deodoro da Fonseca, the first President of Brazil, soon realized that Magalhães was wrong for the post and created the Ministry of Public Instruction and Communication (*Ministério dos Negócios da Instrução Pública, Correios e Telégrafos*) for him to head. As minister, Magalhães implemented educational reforms inspired by positivist philosophy and created a number of short-lived institutes of education. With Magalhães's death in 1891, the first Brazilian Ministry of Education came to an end [46].

The year before his death, Magalhães in his post as minister, wrote to Pierre Lafitte, director of positivism and Auguste Comte's successor in Paris, stating his intention to reform the *Escola militar* along positivistic lines:

> For a long time, I have tried to reconcile my private and public position with the teachings of the founder of the Religion of Humanity, taking into account the particular conditions of the Brazilian environment. I do not suggest that I was never mistaken, or to put it another way, it would be strange if positivism could give the gift of infallibility. I am confident that I disseminated, as best as I could, the true doctrine, and I am willing to continue to serve the cause of Humanity and of my Nation. I will thus take part in the government, despite the cries of those who pretend to be the sole depositories of Auguste Comte's true thought. [40, 645]

Lafitte answered at length, showing the approval of the French positivists of the republic in Brazil and of the fact that an adherent of positivism held a high post in the Brazilian government [46, 646–648].

Although Magalhães's educational reform did not extend to all levels, it did have a decisive influence at the secondary level and, particularly, on the curriculum of the *Colégio Pedro II*. Magalhães tried to put the positivistic ideal into practice; that is, he sought to implement the encyclopedic hierarchy of the positivist sciences into the 7-year course. Thus, students were exposed to the sciences in their Comtian order: mathematics (divided into arithmetic, algebra, geometry, calculus, and mechanics); astronomy; physics; chemistry; biology; and sociology. In addition to these sciences specifically recommended by Comte, linguistics and humanistic subjects were also taught. The great novelty was that calculus, astronomy, and sociology were taught at the secondary level. Moreover, the *Escola normal* also implemented a new curriculum thanks to Magalhães's reform, which incorporated the positivist sciences in Comte's encyclopedic order in its 5-year course [44; 67; 68].

THE EXPANSION OF POSITIVISM IN MATHEMATICS TEXTBOOKS

Undoubtedly, it is in textbooks that the force of positivist ideas can be observed most clearly, and these multiplied in the wake of Magalhães's reform [72]. The conception of mathematics in texts by Magalhães, Raimundo Teixeira Mendes, Samuel Oliveira and Liberato Bitencourt, Licínio Athanasio Cardoso, Roberto Trompowsky de Almeida, and Luis Celestino de Castro clearly reflects Comte's influence. Magalhães, for example,

popularized Comte's *Analytic Geometry* in the military schools [2; 26; 27; 43; 54]; it replaced Sylvestre Lacroix's textbook, which, up to that time, had been the French text preferred by Brazilian teachers [39; 67; 68; 73]. Some students at the *Escola politécnica* of Rio de Janeiro even translated part of Comte's text into Portuguese. As Claudio Costa Ribeiro recalled, "I read Comte. At school we learned analytic geometry with the help of Comte's book. At the beginning of the Republic (1891–1893), all students were either positivists or had some sort of appeal to positivism" [36, 558].

Raimundo Teixeira Mendes was an apostle of the Humanity Church founded in 1881 [40]. In 1887, he wrote a short book, entitled *Elements of Synthetic Geometry*, which demonstrated the strong influence of Comte's ideas [47; 67]. Following Comte, Mendes stated that it was necessary to promote reform at the secondary level. The curriculum should cover the six positive sciences—from mathematics to sociology—in such a way that the method of each science stood out. Relative to geometry, the objective was to organize the teaching so that the relationship between the concrete and the abstract was always emphasized, and time was not wasted on solving particular problems. The idea of dimension, for example, was presented in a way reminiscent of Comte. Mendes started from space and abstracted one dimension at a time to reach the surface, then the line, and then the point.

With the concepts of volume, surface, line, and point thus deduced, Mendes followed Comte in presenting the study of geometry in a true, dogmatic order from the simplest to the more complex. He started with the study of the straight line, then moved to that of surfaces and volumes. Interestingly, the concept of a straight line was not introduced by a definition but through an appeal to visual intuition: "every time we see any point, supposing that from it departs a series of indefinite lines that end at the visual organ of the observer, one will strike us as the shortest one, because the others will represent the involvents in relation to it" [47, 13].

Roberto Trompowsky Leitão de Almeida was born in 1853 in Santa Catarina (Brazil). He attended the *Escola militar* of Rio de Janeiro and later took the military engineering course [40], returning to join the staff of his alma mater in 1877. There, he introduced his students both to the positivist approach and to the wonders of mathematics. One of his students, Alfredo Severo, described his teaching, in these words: "The exciting figure of Colonel Trompowsky appeared in front of us, emerging from the tremendous cruelty of the course's elementary geometry [and] flying high over the stratospheric regions of the transcendental calculus ... . Under his intelligent guidance [and] based on the positive method, the teaching of general geometry emerged from the chaotic confusion characteristic of the dull routine of those popular textbooks that exhibit an obsolete science ruffled only by fastidious and disconnected calculus" [40, 289]. Trompowsky wrote several textbooks [68] and published lectures entitled "algebraic geometry," "differential geometry," and "higher geometry" (all in 1904); "integral geometry" in 1905; and "the rules of false position" in 1923 [1; 2; 3; 4; 5]. In all of these works, which were widely used at military and engineering schools, Comte's influence is evident [9]. In his lectures on algebraic geometry, however, he dealt with analytic geometry, taking an approach different from Comte's. Trompowsky did not separate plane and spatial geometry; he opened with considerations about systems of coordinates in the plane and then moved on to systems of coordinates in space. Unlike Comte, he did not separate theory form its applications; the applications immediately followed the theoretical discussions. Trompowsky's textbooks were widely diffused in teaching at military and engineering school at that time [68].

Samuel de Oliveira and Liberato Bittencourt were Trompowsky's students at the *Escola militar* of Rio de Janeiro in 1890. Due to the lack of an analytic geometry textbook in Portuguese, they decided to produce one. Their book appeared in a first edition in 1892 and in a second in 1895. As they put it in the 1895 preface, "the 1st edition dated 1892 was a success and it was sold out in less than a year ... it was the first time that a book on analytic geometry had been published in Brazil; it was the first time two students had dared to write a book on higher mathematics" [54, x].

Oliveira and Bittencourt took Trompowsky's classes, and so Comte, as their point of departure: "All that is good in our work owes exclusively to Dr. Trompowsky," they stated [54, viii]. Still, "the plan of this book is neither ours, nor Dr. Trompowsky's: it belongs to Auguste Comte, the genial author of *Subjective Synthesis*. Our master was inspired by this brilliant philosopher and, thanks to his enormous talent as an expositor, made it possible for us to produce the plan for the study of Cartesian geometry traced by the great coordinator of the mathematical sciences" [54, ix]. Following Comte, the authors started from geometrical objects and deduced equations [26]. They argued that in order to establish easily the equation of a curved line or surface, one first had to identify the generation of this curve or surface with an explanatory definition [54; 67].

Another author who deserves comment is Licinio Athanasio Cardoso, born in 1852 in Lavras, Rio Grande do Sul [68]. He graduated in mathematics with the title of engineer from the *Escola politécnica* of Rio de Janeiro in 1879 and then earned a medical degree in 1890, focusing on homeopathy. Founder of the School of Medicine and Surgery of Rio de Janeiro and of the Hospital Hahnemanniano, Cardoso also maintained his mathematical interests, writing *Theory of Rotation of Bodies* (1887) and *Elementary Theory of Functions* (1891) and serving as the first President of the Section of Mathematical Sciences of the *Academia Brasileira de Ciências* (founded in 1916).

Cardoso addressed his textbook, *Elementary Theory of Functions*, to the students of the *Escola politécnica* of Rio de Janeiro. In it, he expounded the principles of positivism in the preface and introduction, before proceeding in the next three chapters to present a classification of functions and derivatives, to explore applications, and to lay out notions about differences and interpolation. In particular, he presented the derivative *sui generis*, free from the infinitely small and based on the idea of limit. Rather than treating the derivative as the limit of the relation $\frac{\Delta y}{\Delta x}$, however, he presented it as the state at which the ratio became indeterminate. Cardoso admitted his indebtedness to positivism. In his words, "Comte's opinion is, for us, worth a dogma" [17, 7]. At times, however, Cardoso's positivism seemed to waver. He seemed inclined to reject the strict rules of Comte's system and to formulate his own. He also dared to venture into areas, such as the theory of elliptic functions, that Comte did not treat [17].

Another positivist, Luiz Celestino de Castro, based his 1883 textbook, *Lectures on Arithmetic* (4th ed., 1914), on Comte's method of exposition [21; 27]. This book summarized the lessons he taught at the *Escola Militar* of Rio Grande do Sul, where there were many followers of positivism on the military staff. In it, Celestino de Castro called attention to the fact that it was impossible to follow Comte's plan in its entirety in Brazil because of the "backwardness of our education." Even the most committed Brazilian positivists realized that they could not adhere strictly to Comte's ideals. In 1884, Cardoso, writing of Celestino de Castro's book, acknowledged that the work failed to fulfill the requirements of Comte's

Subjective Synthesis from a purely philosophical point of view: "If your *Arithmetic* met the requirements of the *Subjective Synthesis*, it would not meet its destiny. Only in the institutions and courses where positivist teaching is followed in an orthodox way will it be possible to study arithmetic according to such requirements. However, such teaching, which is destined to produce the greatest and most welcome social reforms, is, unfortunately, not yet done in our academies, and would fail due to the lack of method. Those who intended to teach arithmetic, according to Comte's plan of study requirements, [find] insufficiently prepared individuals, to whom Philosophie Primeira was not taught, as a first scientific support of the mind" [21, 17].

Nevertheless, Celestino de Castro argued that it was possible to follow the natural transition of simple to compound, of easy to difficult, of concrete to abstract. In so doing, the student could go beyond facts to laws and would thereby follow the basic premise of Comte's educational scheme. To this end, Celestino de Castro divided his book into two parts: one devoted to arithmetic, the other to its applications. The first part treated whole numbers and fractions, division, the theory of proportion, and logarithms, while the second part, entitled "practical sociology," involved metrology, the rule of three, interest, and discounts. The work did not differ substantially from other contemporary arithmetics in its mathematical content, but its preface, with its exposition of Comte's conception of mathematics, set it apart philosophically [21].

POSITIVISM IN PERIODICALS

Positivism was also evident in the periodicals that started to appear at the end of the 19th century. The *Revista da Escola politécnica do Rio de Janeiro*, a student initiative undertaken in 1897, was published for about 4 years. In the opening number, Cardoso declared that he would continue to uphold Comte's philosophy and to deny everything that was not in accordance with positivist ideas. One of the school's students also gave written testimony to the influence of Comte's thought, writing in the magazine that "Comte's *Analytic Geometry* is the most beautiful book we know. Through it we have a clear idea of curve generation" [10, 101]. The *Revista da Escola politécnica de São Paulo*, founded in 1904 also owing to a student initiative, showed signs of the influence of positivism, but to a lesser degree than its counterpart in Rio de Janeiro. In its fourth volume, published in 1908, Rodolpho Santiago put forth his ideas on analytic geometry in a series of papers. His articles did not make new contributions to the area, but were rather summaries of Comte's book or of classroom notes [66, 293–309].

At least one other journal appeared outside the context of the military and polytechnic schools. The *Revista brasileira de matemática* was founded and edited by Serebrenick in Rio de Janeiro in 1929 and carried papers of a positivist bent by authors such as Agliberto Xavier. A mathematics teacher at the *Colegio Pedro II*, Xavier wrote, in addition to a number of mathematical textbooks [84; 85; 86], more philosophical articles, such as "Ciência e arte [Science and Art]" in the *Revista* [85]. In the latter, he analyzed the abstract and the concrete in science and art. He aimed to reveal the way in which concrete and abstract reasoning influenced the intellectual development of humanity. He based his analysis on the positive sciences from mathematics to sociology, mentioning Comte and criticizing his opponents such as Joseph Bertrand. He concluded by exalting Comte's ideas on the importance of order and progress: "In both the abstract and the concrete, in the dominion of ideas as in the

social context, progress proceeds faster, from order—even an artificial order—than from disorder" [85, 51–56].

The *Escola politécnica* of Rio de Janeiro, in particular, influenced Rio Grande do Sul through the *Escola militar* of Porto Alegre, where many of its students took up teaching positions. As early as 1880, the *Revista mensal*, a student publication of the *Sociedade científica culto às artes*, was published at the *Escola militar* of Porto Alegre. Contributions to the magazine came from authors of differing beliefs, but several of them were adherents to positivism. By the first decade of the 20th century, two more magazines, *Occidente* (begun in 1906) and *Cruzada* (started in 1908), carried articles by students of the *Escola Militar* that made a strong appeal to positivism [45].

THE GRADUAL DECLINE OF POSITIVISM

While the deputies of the governing board of the *Escola politécnica* of Rio de Janeiro included many followers of Comte, the board was by no means exclusively in the hands of positivists. Thus, when writers such as Otto de Alencar Silva (1874–1912) began pointing out the errors in Comte's mathematics, positivism gradually fell from its dominant position in the engineering school. Indications of this eventual fall had surfaced well before the turn of the century, however [68; 74]. In 1882, for example, deputy Ferreira Viana spoke out strongly against positivism: "The art is dead; the positive philosophy has no inspiration .... [It] comes not only to disturb, to distort, to disfigure, to mutilate the art; it comes to implement the most evil and terrible work, to which I call the attention of the government, primarily responsible for the moral and intellectual destiny of the Empire; it comes to raise the spirit of rebellion of men against all constituted authority, from God, its creator, to the government; it comes to stir the commotion and conflict that disturb our century and to give answers to explain the general discontentment" [40, 254]. That same year, deputy Tarquinio de Souza accused the minister of the empire, Rodolfo Dantas, of being a defender of positivist ideas: "I cannot deny that the positivist tendencies of the Minister, manifested in one of his speeches, affect and confound me .... I cannot fail to harbor serious apprehensions about the positivist tendencies of the noble Minister" [40, 255–256]. The deputy concluded his condemnation, "stating that the Minister's defense of positivism published in the *Diario Official do Brasil*, represented a transgression of the criminal code which prohibited the diffusion by means of printed papers ... of [any] doctrines that directly deny the fundamental truth of God's existence and of the immortality of soul" [40, 258].

Inacio da Cunha Galvão, the head of the *Escola politécnica* of Rio de Janeiro, was also an avowed antipositivist. When orthodox positivists Miguel Lemos and Teixeira Mendes announced that they would give a course on positivist philosophy at the *Escola politécnica* of Rio de Janeiro in 1886, Galvão prevented him from doing so. In an official letter to the minister of the empire, Galvão wrote that "I know that the course they intend to create in the school tends to develop the philosophical doctrine of positivism .... I must say that I consider the dissemination of errors that subvert intelligence and morals more dangerous to humanity than poisons and spoiled food that only destroy or damage the physical organism; since the diffusion of the latter is forbidden in all civilized countries, so the diffusion of the philosophical doctrine of positivism must be forbidden. Relative to the course he proposed to teach at the *Escola politécnica*, I think that, as a state institution, in which the Catholic

religion is recognized in the Constitution, it is not possible to allow a course related to one of the modern, impious doctrines that aims at the destruction of Catholicism."[5]

Yet another critic, Otto de Alencar Silva (1874–1912), was a pioneer Brazilian mathematical researcher. In 1897, at the age of 23, he published a paper, entitled "The Minimal Riemann Surface of a Circular Generatrix," in the *Revista da Escola politécnica* [78]. Like many students of the *Escola Politécnica*, he had been exposed to positivism at the beginning of his academic life, but, as he found errors in Comte's subjective synthesis, he abandoned positivism and started to publish papers against it. In 1898, his article "Some Errors in Auguste Comte's *Subjective Synthesis*" appeared in the *Revista da Escola politécnica* [76]. A modified version of this paper also appeared in the *Journal de ciências matemáticas, físicas e naturais* in Lisbon in 1901. Silva's contact with the Portuguese mathematician, Francisco Gomes Teixeira, moreover helped to familiarized him with contemporary European mathematics. His publications—papers such as "Stokes's Formula" [80], "Geometric Application of Riccati's Equation" [79], and "An Identity in Elliptic Functions" [79]—were characterized by originality and didactic features. Silva taught at the *Escola politécnica* of Rio de Janeiro until his premature death in 1912 [29; 34; 68].

Silva passed on his modern conception of mathematics to two of his best students, Manoel Amoroso Costa and Theodoro Augusto Ramos [77]. Both played an active role in the introduction of new antipositivist, mathematical concepts and theories into Brazil, Costa through his presidency of the Mathematics Section of the Brazilian Academy of Sciences (succeeding Licínio Cardoso) and Ramos through his academic contributions to several important educational institutions in São Paulo [20]. Lélio da Gama (1892–1935) was also a follower of Silva in the new generation of mathematicians that fought to banish positivism from Brazil [15]. The pioneering work of these mathematicians ushered in a new phase in the history of mathematics in the country, but Comte's positivism, although weakened, continued to gather followers for some time, as evidenced by Ivan Lins's analysis of Brazilian academic institutions [40].

Likewise, an interview with Admiral Alfredo Moraes Filho[6] revealed that positivism permeated the *Escola naval* and still had followers as late as the 1920s. Filho, born in 1905, confronted positivism first as an elementary school student around 1910 and then again at the *Colegio Pedro II* and at the *Escola naval* under the direction of Admiral Francisco Machado da Silva. The strength of positivism in Filho's life reflected itself in 1930, when he helped found the still active Positivist Club (*Clube positivista*) [40]. Thereafter, Filho became an orthodox positivist; that is, he joined the Humanity religion and remained both faithful to Comte's principles and a militant for positivism up to 1989, when I interviewed him.

CONCLUSIONS

The mathematics taught in 19th-century Brazilian military and engineering schools as well as Brazilian research works in mathematics—doctoral theses, published papers, textbooks, etc.—were consonant with Brazil's intellectual, social, and governmental contexts. Unlike 19th-century Europe, Brazil's nascent social, economic, and cultural patterns did not generate a need for advanced scientific and technological activities. Since the generalist and

[5] This quote is from a manuscript in the *Arquivo Nacional do Rio de Janeiro*, códice IE[3] 84.

[6] The interview with Admiral Alfredo Moraes Filho was in July 1989.

encyclopedic view of positivist mathematics so perfectly suited the needs of mathematics teachers in Brazil's military and engineering schools, they came to value only those concepts presented by Auguste Comte in his *Cours de philosophie positive* of 1830, a work that well codified 18th- and early 19th-century mathematics [37; 68]. The influence of positivism was even more significant relative to educational policy, where positivist, soldier, politician, and educator Benjamin Constant Magalhães counted among his followers ardent defenders of the republic, who witnessed the end of the imperial era in 1889 and the subsequent beginning of the republican period [78]. Magalhães, as the minister of education, in the new republic, aimed to put into practice his positivist ideology in the 1890 reform of education. In his view, the teaching of mathematics served to promote the positive philosophy and, in so doing, to create a new class of military men who viewed positivism as a way to realize their goals of "order and progress."

Positivism spread and developed in Brazil between 1870 and 1930, mainly owing to the initiatives of a strong leadership within the military and engineering academies. The large number of positivistic mathematics textbooks—more than 40 titles in the period of 1858–1940—testify to the dissemination of Comte's ideas in the student environment.[7] Authors not only taught mathematics but also played major roles in the political and social life of Brazil as ministers, deputies, mayors, governors, engineers, and high-ranking military officials. Today, the positivist phenomenon has yet to be exhausted as a research theme, just as many positivistic concepts and controversies have yet to disappear from Brazilian life.

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